	Sanjay Ghodawat University, Kolhapur Established as State Private University under Govt. of Maharashtra. Act No XL, 2017		2018-19 EXM/P/09/00
Program: B. Sc-III	School of Science <i>section-A</i>	Dept. : Physics	
Course Code: PHS305	Physics VII (Mathematical Physics)	Semester -V	
Saturday, 23 Nov. 2019	Examination: ESE, Max Marks: 100, Time $\frac{1}{2}$ hrs	10:30 AM to 11 AM	
Seat No.:	PRN No.:	Student Sign:	
Invigilator Sign:	Examiner Sign:	Marks Obtained:	

Instructions:

- 1) All Questions are compulsory.
- 2) Mark $\sqrt{\quad}$ to the correct option. Do not circle.
- 3) More than one options marked will not be considered for assessment.
- 4) Rough calculations on paper are not allowed.
- 5) Use of non-programmable calculator is allowed.

Q.1 Select the correct alternative

	Marks	Bloom's Level	CO
1. Which of the following is the property of Matrix Multiplication _____	01	L3	CO1
a) $AB \neq BA$			
b) $A(B+C) = AB+AC$			
c) $A(BC) = (AB)C$			
d) All the above options a, b and c			
2. The square matrix $A = (a_{ij})$ is said to be Symmetric if, _____	01	L1	CO1
a) $A^t = -A$ & $\bar{A} = -A$			
b) $\bar{A} = -A$ & $A' = -A$			
c) $A^t = A$ & $a_{ij} = a_{ji}$			
d) $A^t = -A$ & $a_{ii} = 0$			
3. A system of homogeneous linear equations $AX = O$ is said to be consistent and has unique solution if,	01	L1	CO1
a) $R(A) = n$			
b) $R(A) \neq R(C)$			
c) $R(A) = R(C) = n$			
d) $R(A) < n$			
4. The averaging or mean operator, μ , is defined by _____	01	L1	CO2
a) $\mu = \frac{1}{2} (E^{1/2} - E^{-1/2})$			
b) $\mu = \frac{1}{2} (E^{1/2} + EE^{-1/2})$			
c) $\mu = \frac{1}{2} (EE^{1/2} - E^{-1/2})$			
d) $\mu = \frac{1}{2} (E^{1/2} + E^{-1/2})$			
5. The first order difference of $f(x)$ in the forward difference is defined as	01	L2	CO2
a) $\Delta f_n = f(x_{n+1}) + f(x_n)$			
b) $\Delta f_n = f(x_{n+1}) - f(x_n)$			
c) $\Delta f_n = f(x_{n-1}) - f(x_n)$			
d) None of the above			

6. The first order difference of $f(x)$ in the backward difference is defined as
- a) $\Delta f_n = f(x_n) - f(x_{n-1})$ b) $\nabla f_n = f(x_n) - f(x_{n-1})$
c) $\nabla f_n = f(x_n) + f(x_{n-1})$ d) $\Delta f_n = f(x_n) - f(x_{n+1})$
7. Which of the following is not a beta function_____
- a) $\beta(l, m) = \int_0^1 x^{l-1} (1-x)^{m-1} dx$ b) $\beta(m, l) = \int_0^1 x^{m-1} (1-x)^{l-1} dx$
c) $\beta(l, m) = \beta(m, l)$ d) $\beta(m, l) = \int_0^1 x^{m-1} (1+x)^{l-1} dx$
8. The relation between beta and gamma function is given by
- a) $\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$ b) $\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l-m)}$
c) $\beta(m, l) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l-m)}$ d) None of the above
9. The complementary error function of x , $\text{erfc}(x)$ is
- a) $\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ b) $\frac{\sqrt{\pi}}{2} \int_{-\infty}^x e^{-t^2} dt$
c) $\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{t^2} dt$ d) $\frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$
10. Which of the following is not correct
- a) $\overline{n+1} = n\overline{n}$ b) $\overline{n+1} = n!$
c) $\overline{1} = 1$ d) None of the above
11. The error function of ∞ , $\text{erf}(\infty)$ is
- a) 2 b) 1
c) $\sqrt{\pi}$ d) 0
12. The error function of $\text{erf}(-x)$ is
- a) $-\text{erf}(x) + \text{erf}(-x)$ b) $\sqrt{\pi}$
c) $-\text{erf}(x)$ d) ϵ
13. $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ is _____
- a) π b) $\sqrt{\pi}$
c) 2 d) $2\sqrt{\pi}$
14. Which of the following function is even?
- a) $f(x) = x$ b) $f(x) = x^2$

c) $f(x) = x^3$

d) $f(x) = \sin x$

15. The Fourier coefficient b_n for $n = 1, 2, 3, \dots$ can be represented by _____ 01 L1 CO4

a) $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin^2 nx dx$

b) $b_n = \frac{1}{2} \int_{-\pi}^{\pi} f(x) \cos nx dx$

c) $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$

d) $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$

16. The Fourier for square wave for the function $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$ can be written as _____ 01 L1 CO4

a) $f(x) = \frac{h}{4} + \frac{2h}{\pi} \left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right]$

b) $f(x) = \frac{h}{4} + \frac{\pi}{2h} \left[\sin x + \frac{\sin 5x}{5} + \dots \right]$

c) $f(x) = \frac{h}{4} + \frac{\pi}{2h} \left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right]$

d) $f(x) = \frac{h}{4} + \frac{\pi}{2h} \left[\sin x + \frac{\cos 5x}{5} + \dots \right]$

17. The expansion of Fourier series is _____ 01 L1 CO4

a) $f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \frac{\cos n\pi x}{L} + b_n \frac{\sin n\pi x}{L} \right)$

b) $f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \frac{\cos n\pi x}{L} + b_n \frac{\cos n\pi x}{L} \right)$

c) $f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \frac{\sin n\pi x}{L} - b_n \frac{\sin n\pi x}{L} \right)$

d) $f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \frac{\sin n\pi x}{L} + b_n \frac{\cos n\pi x}{L} \right)$

18. The Fourier coefficient a_n can be represented by _____ 01 L1 CO4

a) $a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$

b) $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$

c) $a_n = \frac{1}{2} \int_{-\pi}^{\pi} f(x) \sin nx dx$

d) $a_n = \frac{1}{2} \int_{-\pi}^{\pi} f(x) \sin^2 nx dx$

19. The Fourier for sawtooth wave can be written as _____ 01 L1 CO4

a) $(y_t) = a + \frac{a}{\pi} [\sin \omega t + \frac{1}{2} \cos 2\omega t + \frac{1}{3} \sin \omega t + \dots]$

b) $(y_t) = a + \frac{a}{\pi} [\sin \omega t + \frac{1}{3} \sin \omega t + \dots]$

c) $(y_t) = a + \frac{a}{\pi} [\sin \omega t + \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t + \dots]$

d) $(y_t) = a + \frac{a}{\pi} [\sin \omega t + \frac{1}{2} \sin \omega t + \dots]$

20. The Fourier for full wave rectifier for the function $f(x) = \begin{cases} \sin \omega t & 0 < \omega t < \pi \\ -\sin \omega t & -\pi < \omega t < 0 \end{cases}$ can be written as

01

L1


CO4

a) $f(t) = \frac{4}{\pi} + \frac{4}{\pi} \left[\frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \dots \right]$

b) $f(t) = \frac{4}{\pi} + \left[\frac{\cos 2\omega t}{3} + \frac{\sin 4\omega t}{5} + \dots \right]$

c) $f(t) = \frac{4}{\pi} - \frac{4}{\pi} \left[\frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \dots \right]$

d) $f(t) = \frac{4}{\pi} + \left[\frac{\cos 2\omega t}{3} + \frac{\sin \omega t}{25} + \dots \right]$

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Q.2 Attempt the following questions.

- | | | | | |
|----|---|----|----|-----|
| a) | The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ Find the eigen values of | 08 | L3 | CO1 |
|----|---|----|----|-----|

$$3A^3 + 5A^2 - 6A + 2I.$$

- | | | | | |
|----|---|----|----|-----|
| b) | If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 2 & 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ find the product AB and BA, | 04 | L2 | CO1 |
|----|---|----|----|-----|

and show that $AB \neq BA$.

OR

- | | | | | |
|----|--|----|----|-----|
| b) | Determine the values of α, β , and γ when $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is | 04 | L2 | CO1 |
|----|--|----|----|-----|

orthogonal.

Q.3 Attempt the following questions.

- | | | | | |
|----|--|----|----|-----|
| a) | Using Simpson's 1/3 Rule, find the value of the integral $\int_0^1 \frac{dx}{1+x}$ correct | 08 | L2 | CO2 |
|----|--|----|----|-----|

to third decimal place. {Take $h = 0.25$ }

- | | | | | |
|----|---|----|----|-----|
| b) | Write the expressions for first order and second order differentiation using <i>Stirling Formula</i> for $x = x_0 + rh$ and $x = x_0$. | 04 | L3 | CO2 |
|----|---|----|----|-----|

OR

- | | | | | |
|----|--|----|----|-----|
| b) | Derive an expression for <i>Central difference Interpolation Formula</i> . | 04 | L3 | CO2 |
|----|--|----|----|-----|

Q.4 Attempt the following questions.

Marks	Bloom's Level	CO
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|----|------------|----|----|-----|
| a) | Show that, | 08 | L4 | CO3 |
|----|------------|----|----|-----|

$$1. \beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$$

2. $\beta(l, m) = \beta(m, l)$ using beta and gamma function.

- | | | | | |
|----|------------------------------|----|----|-----|
| b) | Evaluate using beta function | 08 | L5 | CO3 |
|----|------------------------------|----|----|-----|

$$1. \int_0^1 x^4 (1-\sqrt{x})^5 dx \quad 2. \int_0^1 (1-x^3)^{\frac{1}{2}} dx$$

- | | | | | |
|----|---|----|----|-----|
| c) | Evaluate $\int_0^\infty \frac{x^a}{a^x} dx$ using gamma function, hence show that | 04 | L3 | CO3 |
|----|---|----|----|-----|

$$\int_0^\infty \frac{x^7}{7^x} dx = \frac{7!}{(\log 7)^8}$$

- | | | | | |
|----|--|----|----|-----|
| d) | Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n}{(m+1)^{n+1}} \Gamma(n+1)$ | 04 | L5 | CO3 |
|----|--|----|----|-----|

- | | | | | |
|----|---|----|----|-----|
| e) | Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin \theta}} d\theta = \pi$ | 04 | L2 | CO3 |
|----|---|----|----|-----|

OR

- | | | | | |
|----|--|----|----|-----|
| e) | Show that $\int_0^\infty e^{-x^2-2bx} dx = \frac{\sqrt{\pi}}{2} e^{b^2} [1 - \operatorname{erf}(b)]$ | 04 | L2 | CO3 |
|----|--|----|----|-----|

Q.5 Attempt the following questions.

Marks	Bloom's Level	CO
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- | | | | | |
|----|--|----|----|-----|
| a) | Find the Fourier series for $f(x) = x^2$ in the interval $[-\pi, \pi]$. | 08 | L3 | CO4 |
| b) | Find the Fourier series for $f(x) = x $ in the interval $[-\pi, \pi]$. | 08 | L2 | CO4 |

- | | | | | |
|-----------|---|----|----|-----|
| c) | Find the Fourier integral | 04 | L2 | CO4 |
| | $f(x) = \begin{cases} 1 & x < 1 \\ 0 & x > 1 \end{cases}$ | | | |
| d) | Elaborate sine Fourier series. | 04 | L2 | CO4 |
| e) | Obtain Fourier series expansion for full wave rectifier. | 04 | L3 | CO4 |
| OR | | | | |
| e) | Expand Fourier series for sawtooth wave. | 04 | L3 | CO4 |