Sanjay Ghodawat University, Kolhapur
Established as State Private University under Govt. of Maharashtra.
Act No XL, 2017

2019-20
EXM/P/09/00

Year and Program:2019-20 B.Sc.III	**School of Science**	**Department of Mathematics**
Course Code – MTS 301	**Course Title – Metric Space**	**Semester – V**
Day and Date – Tuesday 19-11-2019	**End Semester Examination**	**Time:** 10.30 to 11.00 am.
		Max Marks: 100
PRN number –	**Seat no-**	**Answer Booklet No.-**
Students' Signature -	Section - A.	**Invigilator's Signature –**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

		Marks	Bloom's Level	CO
Q.1	**Tick the correct answers.**			
i)	Which one of the following is a metric on $\mathbb{R}$? A) $d(x, y)=\lvert x^{40}-y^{40}\rvert$ B) $d(x, y)=\lvert x^{7}-y^{7}\rvert$ C) $d(x, y)=\max(2x, y)$ D) $d(x, y)=\min(x, 2y)$	02	L_3	CO1
ii)	Consider the following two statements I) Arbitrary union of open set is open. II) Arbitrary intersection of open set is open. Then A) only I is true, B) only II is true , C) both I and II are true , D) both I and II are false.	02	L_2	CO2
iii)	Which one of the following set is not connected in $\mathbb{R}$? A) [0,2], B)) $[0,2]\cup[1,3]$, C) $[0,2]\cup[3,4]$, D) All A,B,C.	02	L_2	CO3

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iv)	A subset A of R_d is totally bounded if and only if A contains A) finite number of points, B) infinite number of points, C) only one point, D) At most two point.	02	L_1	CO4
v)	Closed Subset of complete metric space is A) compact, B) complete, C) connected, D) totally bounded.	02	L_2	CO4
vi)	If T is a contraction on M then which one of the following is correct? $A)\rho(T_x,T_y)\le\frac{5}{4}\rho(x,y),\quad B)\rho(T_x,T_y)\le\frac{4}{3}\rho(x,y),$ $C)\rho(T_x,T_y)\le\frac{1}{2}\rho(x,y),\quad D)\rho(T_x,T_y)\le\frac{22}{7}\rho(x,y).$	02	L_3	CO4
vii)	A metric space $<M,\rho>$ is said to be compact if it is A) complete and totally bounded, B) complete and bounded, C) open and bounded, D) open and totally bounded.	02	L_2	CO5
viii)	Consider the following two statements I) A function continuous on a compact domain is uniformly continuous. II) A function continuous on a connected domain is not uniformly continuous. A) only I is true, B) only II is true , C) both I and II are true , D) both I and II are false	02	L_2	CO5
ix)	Consider the following two statements I) The continuous image of compact set is compact. II) The continuous image of complete set is compact A) only I is true, B) only II is true , C) both I and II are true , D) both I and II are false	02	L_2	CO5
x)	Every sequence of points in a metric space M has a subsequence converging to appoint in M then M is A) compact B) complete C) connected D) totally bounded	02	L_1	CO5

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2019-20

EXM/P/09/01

Year and Program:2019-20 **School of Science** **Department: Mathematics**

B.Sc. III

Course Code: MTS301 **Course Title -Metric Space** **Semester – V**

Day and Date Tuesday 19-11-2019 **End Semester Examination** **Time:** 11.00 am to 1.30 pm. **Max Marks: 100**

Instructions:

1) All questions are compulsory.
2) Use of non-programmable calculator is allowed.
3) Figures to the right indicate full marks.

Section- B

			Marks	Bloom's Level	CO
Q.2		**Attempt any two of the following.**			
	a)	In a metric space show that limit of a sequence is unique.	06	L_4	CO1
	b)	If ρ and σ are two metrics defined on M then show that $\rho+\sigma$ is also a metric space on M.	06	L_3	CO1
	c)	Let $< M, \rho >$ be a metric space, a$\in M$ and f, g be real valued functions defined on M. If $\lim_{x\to a} f(x)$=L and $\lim_{x\to a} g(x)$=N then show that $\lim_{x\to a}(f(x)\, g(x)) = LN$.	06	L_3	CO1
Q.3		**Attempt any two of the following.**			
	a)	Define an open set. Show that in any metric space every open ball is an open set.	07	L_4	CO2
	b)	Define limit point of a set. Show that finite union of closed sets is closed.	07	L_4	CO2
	c)	Show that F is continuous iff inverse image of a closed set is closed.	07	L_3	CO2
Q.4		**Attempt any two of the following.**			
	a)	Define connected set. Let $f : M_1 \to M_2$ be a continuous function. If M_1 is connected then show that $f(M_1)$ is also connected.	07	L_3	CO3
	b)	If $A \subseteq M$ is totally bounded set then show that A is bounded.	07	L_3	CO3
	c)	Give an example of an infinite subset of l^2 which is bounded but not totally bounded.	07	L_3	CO3
Q.5					
	a)	Define I) Complete metric space. II) Contraction Operator.	04	L_2	CO4

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	b)	**Attempt any two of the following.**			
	i)	Prove that $\mathbb{R}_d$ is complete.	08	L_3	CO4
	ii)	Let $< M, \rho >$ be a complete metric space. for each $n \in I$, and F_n is a closed bounded subset of M such that $a) F_1 \supset F_2 \supset ... \supset F_n \supset ...$ $b) diam\, F_n \to 0\ as\, n \to \infty$ then show that $\bigcap_{n=1}^{\infty} F_n$ contains exactly one point.	08	L_4	CO4
	iii)	If T is a function $T:[0,1] \to [0,1]$ and if there exists a real number α with $0 \le \alpha < 1$ such that $\left\| T_x' \right\| \le \alpha$ where T' is derivative of T, prove that T is contraction on $[0,1]$.	08	L_3	CO4
Q.6		**Attempt any two of the following.**			
	a)	If the metric space M has Heine-Borel property, then show that M is compact.	10	L_5	CO5
	b)	Let $<M_1, \rho_1>$ be a compact metric space if f is continuous function from M_1 into a metric space $<M_2, \rho_2>$ then show that f is uniformly continuous on M_1.	10	L_4	CO5
	c)	Let f be a continuous function from a compact metric space M_1 into M_2. Show that $f(M_1)$ is compact. Also show that any finite subset of every metric space is compact.	10	L_3	CO5

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