SGU

Sanjay Ghodawat University, Kolhapur

2019-20
EXM/P/09/00

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

Year and Program: 2019-20 T.Y.B.Sc **School of Science** **Department: Mathematics**

Course Code – MTS 303 **Course Title – Linear Algebra** **Semester – V**

Day and Date – Thursday 21/11/19. **End Semester Examination** **Time:** 1/2 hr [10.30 am to 11 am]

Max Marks: 100

PRN number – **Seat no-** **Answer Booklet No.-**

Section A.

Students' Signature - **Invigilator's Signature –**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

		Marks	Bloom's Level	CO
Q.1	Tick mark (√) the correct alternative			
i)	Let V be a vector space over F, then which one of the following statements is true? a) If W_1 and W_2 be two subspaces of V then so does $W_1 \cup W_2$. b) If W is subspace of V then $\dim V \leq \dim W$. c) Any linearly independent set of V can be extended to form a basis of V. d) For any $S \subseteq V$, then $L(S)$ is the largest subspace of V containing S.	2	L1	CO1
ii)	If $T: R^3 \to R^3$ is the linear transformation such that $T(x, y, z) = (x+z,\ x+y+z,\ 2x+y+3z)$ then a) $\dim range(T) = 1$, c) $\dim range(T) = 2$, b) $\dim Ker(T) = 0$, d) $\dim Ker(T) = 3$.	2	L4	CO2

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iii)	Rank of the matrix $A=\begin{bmatrix}6&1&3&8\\4&2&6&-1\\10&3&9&7\\16&4&12&15\end{bmatrix}$ is a) 1, b) 2, c) 3, d) 4.	2	L3	CO3
iv)	If $\hat{V}$ is a dual basis of V then a) $\dim V=\dim\hat{V}$, c) $\dim V\le\dim\hat{V}$, b) $\dim V+\dim\hat{V}=0$, d) $\dim V\ge\dim\hat{V}$.	2	L1	CO4
v)	Let the matrix $A=\begin{pmatrix}1&2\\-2&1\end{pmatrix}$ and I be an identity matrix of order 2, then $A^2-2A+5I$ is a) $\begin{pmatrix}1&0\\0&1\end{pmatrix}$, c) $\begin{pmatrix}1&1\\1&1\end{pmatrix}$, b) $\begin{pmatrix}0&0\\0&0\end{pmatrix}$, d) $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$.	2	L3	CO4
vi)	The eigen values of matrix $A=\begin{pmatrix}1&2\\1&0\end{pmatrix}$ are a) 1, 2 b) −1, 2 c) 1, −2 d) −1, −2	2	L3	CO4
vii)	If 1, 2 and 5 are eigen values of the matrix A, then the eigen values of the $adj(A)$ are a) 1, 2 ,5 b) $1, \frac{1}{2}, \frac{1}{5}$ c) 2, 5, 10 d) 1, 4, 25	2	L3	CO4
viii)	For the matrix $A=\begin{bmatrix}2&-2&3\\-2&-1&6\\1&2&0\end{bmatrix}$, one of the eigen value of A is 3 then other two eigen values are a) 2, −5 b) 3, -5 c) 3, −5 d) 3, 5	2	L3	CO5

ix)	If u and v are the elements of an inner product space V and $\|u\| = 4, \|u+v\| = 2, \|u-v\| = 6$ then a) $\|v\| = 2,$ b) $\|v\| = \sqrt{2},$ c) $\|v\| = 4,$ d) $\|v\| = \sqrt{4}.$	2	L2	CO5
x)	Which one of the following sets of vectors is orthogonal? a) $\{(3,0,4),(-4,0,3),(0,1,0)\},$ b) $\{(3,0,4),(-1,0,7),(2,9,11)\},$ c) $\{(1,0,1),(1,0,-1),(0,3,4)\},$ d) $\{(1,0,1),(1,1,-1),(0,3,4)\}.$	2	L3	CO5

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Year and Program: 2019-20 **School of Science** **Department of Mathematics**
T.Y.B.Sc
Course Code: MTS 303 **Course Title: Linear Algebra** **Semester – V**
Day and Date: Thursday 21/11/19 **End Semester Examination (ESE)** Section B. **Time:** 2.5hrs [11am to 1.30pm]
Max Marks: 100

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Non-programmable calculator is allowed
4)R: Set of real numbers.

			Marks	Bloom's Level	CO
Q.2		**Solve any Two of the following.**			
	i)	Show that intersection of two subspaces is a subspace.	6	L1	CO1
	ii)	If V is a finite dimensional vector space and $\{v_1, v_2, v_3, \cdots v_r\}$ is a linearly independent subset of V, then show that it can be extended to form a basis of V.	6	L1	CO1
	iii)	Show that the vectors $(1,0,0,0),(1,1,0,0),(0,0,0,1),(1,1,1,1)$ in $R^4(R)$ are linear independent.	6	L3	CO1
Q.3		**Solve any Two of the following.**			
	i)	Let $T: V \to W$ be a linear transformation, then show that $\dim V = Rank\,T + Nullity\,T$.	7	L1	CO2
	ii)	Let $T: V \to U$ be a linear transformation, then show that $\frac{V}{Ker\,T} \cong Range\,T$.	7	L1	CO2
	iii)	Find the range, rank, kernel and nullity of the linear transformation $T: R^2 \to R^3$ defined by $T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$.	7	L2	CO2

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			Marks	Level	CO
Q.4		**Solve any Two of the following.**			
	i)	Let T be a linear operator on R^3, defined by $T(x_1, x_2, x_3) = (2x_1,\ x_1 - x_2,\ 5x_1 + 4x_2 + x_3)$. Show that T is invertible and find T^{-1}.	7	L3	CO3
	ii)	Find the rank of the matrix $A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 8 & 9 & 10 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$.	7	L3	CO3
	iii)	Let $T : V \to W$ and $S : W \to U$ be two linear transformations then show that i) If S and T are one-one, onto then ST is one-one, onto and $(ST)^{-1} = T^{-1}S^{-1}$. ii) If ST is one-one then T is one-one.	7	L1	CO3
Q.5		**Solve any Four of the following.**			
	i)	Let T be a linear operator on a finite dimensional vector space V over F. Then show that $c \in F$ is an eigen value of T if and only if $T - cI$ is singular.	5	L2	CO4
	ii)	Define similar matrices. Show that similar matrices have same characteristic polynomial.	5	L2	CO4
	iii)	Obtain the eigen values, eigen vectors of $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.	5	L3	CO4
	iv)	Verify Caley-Hamilton theorem for the matrix A, where $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.	5	L2	CO4
	v)	Let v and w be eigen vectors of T corresponding to two distinct eigen values of T. Show that $v + w$ cannot be an eigen vector of T.	5	L2	CO4

Q.6	**Solve any Four of the following.**			
i)	Let V be an inner product space. Show that $\|\langle u, v\rangle\| \leq \|u\|\|v\|$ for all $u, v \in V$.	5	L1	CO5
ii)	Let V be an inner product space. Then show that $\|x+y\|^2+\|x-y\|^2=2\left(\|x\|^2+\|y\|^2\right)$ for all $x, y \in V$.	5	L2	CO5
iii)	Let S be an orthogonal set of non-zero vectors in an inner product space V. Then show that S is a linearly independent set.	5	L2	CO5
iv)	Obtain an orthonormal basis with respect to the standard inner product for the subspace of R^3 generated by $(1,0,3)$ and $(2,1,1)$.	5	L4	CO5
v)	If V is a finite dimensional inner product space and W is a subspace of V, then show that $V=W \oplus W^{\perp}$.	5	L1	CO5
vi)	Let $u=(x_1, x_2)$ and $v=(y_1, y_2) \in \mathbf{R}^2$. Verify that $\langle u, v\rangle = x_1 y_1 - 3x_1 y_2 - 3x_2 y_1 + 10x_2 y_2$ is an inner product on R^2.	5	L4	CO5

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